Analytical methods and improved non-equilibrium droplet evaporation models for high pressure conditions

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Seminar organised within the framework of DROPIT

11th and 12th of July, 2019
University of Stuttgart, ITLR,
Pfaffenwaldring 31, Room 2.222

Schedule

Unit 1 - Lecturer G.E. Cossali: 11.07.2019 from 09:30 till 12:00
Unit 2 - Lecturer S. Tonini: 11.07.2019 from 14:00 till 16:30
Unit 3 - Lecturer G. Lamanna: 12.07.2019 from 09:30 till 12:00

Abstract

The seminar is organized in three separates lecture units, presenting different approaches to the modelling of droplet evaporation. A short description of the specific lecture unit can be found below.

Unit 1: Orthogonal functions and generalised Fourier series

G.E. Cossali

Boilly, Julien-Leopold. (1820). Album de 73 Portraits-Charge Aquarelle's des Membres de l'Institute (watercolor portrait #29). Biliotheque de l'Institut de France.

The concept of orthogonality comes from geometry as a generalisation of the concept of perpendicularity, which is the property of two lines that intersect at a right angle. Two vectors are said to be orthogonal when they form a right angle and in vector algebra formalism this property can be simply expressed by saying that the scalar product (the dot product) of the two vectors is nil. The orthogonality among functions is a property that can be expressed as a generalisation of this last definition. Orthogonal functions arise naturally in many fields of applied mathematics. In many physical and engineering problems solutions of PDEs often lead to Sturm-Liouville (SL) problems and the solution of such problems is obtained in terms of orthogonal functions; approximation of function is often efficiently performed using set of orthogonal functions; spectral decomposition of signals and fields relies on the use of orthogonal functions. The lecture will give an introduction to the properties and use of orthogonal functions in engineering problems. Elements of functional analysis, such as linear spaces, inner product, Hilbert spaces, etc. will be used as a basis to introduce the general notion of orthogonality among functions. Concepts like projection over subspaces, norm and distance will be discussed and employed to develop methods for approximating functions through linear combinations of orthonormal functions. The generalised Fourier series will be also briefly treated. Examples of application to decomposition of surfaces will be presented with reference to oscillating droplets.

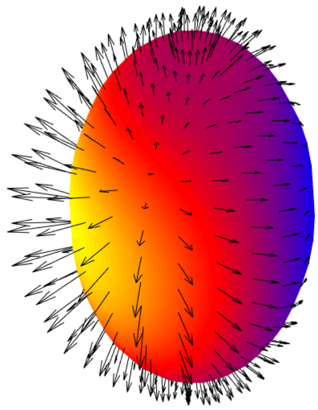
Time schedule of the seminar:

1. Elements of functional analysis (spaces, inner product, norm, etc.) (1h)
2. Orthogonal functions, sequences of orthonormal functions, generalised Fourier series (1h)
3. Example of application to deforming spheroidal drops (1/2h)

Unit 2: Analytical modelling of heating and evaporation of liquid drops

S. Tonini

Analytical modelling of heating and evaporation of drops in gaseous environment is usually tackled starting from the species, energy and momentum conservation equations and introducing simplifying assumptions to yield a set of differential equations that may admit analytical solutions. The lecture will present a method to solve the set of non-linear PDEs that describe the quasi-steady transport through the gas phase of energy and species from an evaporating drop of general shape. The approach that will be presented takes into account the effect of temperature gradients in the gas phase, and the analytical solution of the problem is obtained assuming the temperature dependence of the gas mixture thermo-physical properties (mixture density, thermal conductivity, specific heat and diffusion coefficients). Since the method is in principle applicable to any drop shape, the proposed analytical solution can be used to model heating and evaporation for various drop shapes and geometrical configurations by means of appropriate curvilinear coordinate systems. The effect of non-uniform drop surface temperature will also be addressed for spherical and spheroidal drops.



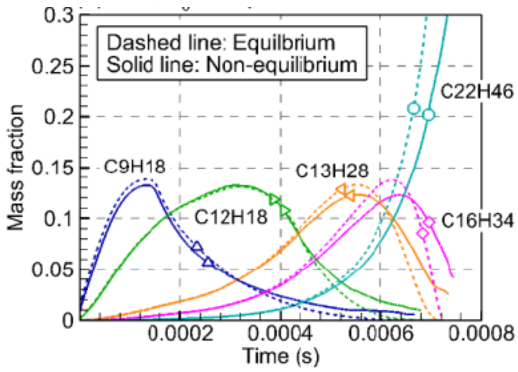
Vapour fluxes in prolate drop with non-uniform surface temperature.

Time schedule of the seminars:

1. Analytical modelling of quasi-steady evaporation in high temperature environments (1h)
2. Effect of drop shape and geometrical configuration (1 h)
3. Effect of non-uniform boundary conditions (1/2 h)

Unit 3: Improved non-equilibrium droplet evaporation models for high pressure conditions

G. Lamanna



Evaluation of the non-equilibrium contribution to the evaporation of multi-component droplets

In this seminar, we will present improved non-equilibrium evaporation models with respect to their applications to high-pressure, high temperature conditions. A first example of their application to multi-components mixtures will be also discussed. The objective of the course is to highlight when non-equilibrium effects become important with respect to realistic engineering applications. The course is structured as follow. First, the choice of appropriate boundary conditions will be discussed in connection to the validity of the local equilibrium assumption at the droplet interface. Second, the importance of including solubility effects in the liquid phase will be discussed. Third, the effect of evaporative cooling on the broadening of the liquid/vapour interface at high pressure condition will be discussed. Finally, two non-equilibrium models will be introduced: the Young and the Bellan model, respectively, together with the discussion of a few sample solutions. The results from evaporation models will be compared either to experiments or to molecular dynamic simulations.